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Width of the Human Visual Spread Function as Determined Psychometrically*

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The response of the human visual system to an optical image is assumed to be linearly related to the logarithm of the spread function of the photographic system projected onto the retina combined with the spread function of the visual system. From psychophysical data derived from viewing (at different distances) a series of pictures generated with different spread functions, an estimate is obtained of the variance of the spread function of the visual system. The square root of this variance ranges from $3\,\mu$ to $8\,\mu$, depending on the techniques used and on the training of the judges. Although the residual errors in this determination are small, they show systematic trends, indicating that definition depends on other factors than the composite variance.

A PICTURE can be produced by either a painter or a photographer. The painter's tool is a brush. If he wishes to show fine detail, he uses a fine brush, but if he considers the broad outlines of the objects to be more important, he uses a broad brush. The photographer's tool, however, is the composite spread function $f_P(x)$ of his photographic system, which consists of the combination of all its elemental spread functions, e.g., that of the camera lens, the negative film, the enlarger lens, the positive film, and, in the case of systems having television-type links, the pickup tube, etc. Like the painter, he can control the size of his brush—his spread function—and, to some degree, its shape.

The stimulus received by the higher receptor center in the brain of an observer looking at the picture is determined by the spread function of the physical system $f_P(x)$ combined in some manner with that of the visual system $f_V(x)$ to give $f_{PV}(x)$. It is assumed in this paper that $f_V(x)$ may be convoluted with $f_P(x)$.

The *visual spread function* $f_V(x)$ likewise comprises several elemental spread functions, e.g., that of the eye lens, the ocular media, the retina, and the higher receptor centers. The visual spread function represents not merely the physical spread function of the eye, but rather the spread function of the entire visual mechanism, objective and subjective.

A major aspect of this subjective response is definition or detail reproduction. In other words, if an observer looks at two pictures, A and B, of identical subject matter, and if $f_{PV}(x)$ is narrower for A than for B, then the observer, on the average, will state that A has better definition than B. For a quantitative study, both the subjective and the physical aspects must be specified numerically.

I. HISTORY

Wolfe and Eisen[1] showed that the definition[2] of pictures could be specified psychometrically with the use of techniques described by Thurstone[3] and Guilford.[4]

Higgins and Jones[5] showed that a quantity termed "acutance" correlated with picture definition under conditions in which the visual spread function could be neglected.

Morrissey[6] developed a mathematical technique for obtaining psychometric values of picture definition by using incomplete paired-comparison data.

Using Morrissey's technique, Higgins and Wolfe[7] showed that, in a more general case, acutance data would predict definition if the resolving power of the eye was taken into account. In other words, $f_{PV}(x)$ was required to predict definition.

Higgins, Lamberts, and Wolfe[8] obtained good correlation between picture definition and the equivalent passband derived from the sine-wave response of the physical system, sine-wave response being the Fourier transform of $f_P(x)$. This relationship was nonlinear, indicating that some other factor, possibly $f_V(x)$, was playing a significant part in determining definition. In these studies, granularity was either negligible or was held constant throughout the experiment so that the results were independent of the granular structure of the photographic materials.

Wolfe and Tuccio[9] studied the detail rendition in a photographic reproduction, first made by a stationary camera and then by a moving camera. In the latter case, a spread function representing the motion of the

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1 R. N. Wolfe and F. C. Eisen, J. Opt. Soc. Am. 43, 914 (1953).

2 The term "sharpness" was used for this concept by them, but that term has since been reserved for a description of the appearance of a single edge.

3 L. L. Thurstone, Am. J. Psychol. 38, 368 (1927).

4 J. P. Guilford, *Psychometric Methods* (McGraw-Hill Book Company, Inc., New York, 1954), pp. 154-177.

5 G. C. Higgins and L. A. Jones, J. Soc. Motion Picture Television Engrs. 58, 277 (1952); PSA Journal (Phot. Sci. Tech.) 19B, 55 (1953).

6 J. H. Morrissey, J. Opt. Soc. Am. 45, 373 (1955).

7 G. C. Higgins, and R. N. Wolfe, J. Opt. Soc. Am. 45, 121 (1955).

8 G. C. Higgins, R. L. Lamberts, and R. N. Wolfe, Optica Acta 6, 272 (1959).

9 R. N. Wolfe and S. A. Tuccio, Phot. Sci. Eng. 4, 330 (1960).

camera had to be included. The spread function $f_P(x)$ of the lens-film combination was represented by the dispersion σ_B of a normal distribution curve having the same area under it as the spread function itself. Detail rendition was found to be proportional to the product of σ_B and the dispersion σ_D of the granularity fluctuations in the reproduction. This result agrees with the theoretical deductions of Zweig, Higgins, and MacAdam.[10]

In the experiment just described, the photographs presented to the observers were 30× to 400× enlargements of the original negatives and were viewed at the normal viewing distance. Consequently, the spread function $f_V(x)$ of the eye was small relative to $f_P(x)$ and could be neglected.

Lowry and DePalma[11] have determined a sine-wave response of the human visual system by obtaining photometric data on the subjective Mach phenomenon. Their data give information[12] about the visual spread function.

As has been pointed out earlier, the results of some of the experiments described in this section were influenced by the visual spread function. It should therefore be possible to make an estimate of the size of this spread function by analyzing data obtained from these and from similar experiments. Data already presented by Higgins, Lamberts, and Wolfe[8] and by Wolfe,[13] along with additional data, have been so analyzed, and the aim of the present paper is to give these results.

That it should be possible to make such an analysis has been indicated by Selwyn[14] and by Schade,[15] since each of them included visual-response data in their over-all analyses of the performance of optical systems.

II. THEORY

The way in which the physical spread function $f_P(x)$ and the visual spread function $f_V(x)$ can be combined to give the function $f_{PV}(x)$ will now be considered in detail, and a mathematical model relating subjective definition to these factors will be proposed.

The spread functions are, for simplicity, taken as the line spread functions, i.e., the line integrals of the corresponding point spread functions.

It is assumed that these functions can be convoluted. The spread function obtained by convoluting $f_P(x)$ and $f_V(x)$ is then

$$f_{PV}(z)=\int_{-\infty}^{+\infty} f_P(x)f_V(z-x)dx. \qquad (1)$$

It will be desirable to specify the half-width of the spread functions by a single number. If $f(x)$ is symmetrical about $x=0$ and is normalized, the half-width is taken as

$$\sigma=\left[\int_{-\infty}^{\infty} x^2 f(x)dx\right]^{\frac{1}{2}}. \qquad (2)$$

Borrowing from statistical terminology, we may call σ the *dispersion* or *standard deviation* of x, and σ^2, which is the second moment of x, we may call the *variance*. Since it can be shown[16] that variances are additive, we can write

$$\sigma_{PV}^2=\sigma_P^2+\sigma_V^2. \qquad (3)$$

These values of standard deviation σ only serve to specify the half-widths of the various spread functions and give no information about their shapes.

A mathematical model relating subjective definition to the physical and visual spread functions will now be proposed. Let us assume that picture definition is linearly related to $\log\sigma_{PV}^2$. We can then write

$$r=a+b\log\sigma_{PV}^2, \qquad (4)$$

where r is the subjective visual response to a picture in terms of definition or detail rendition. Making use of Eq. (3), we can write

$$r=a+b\log(\sigma_P^2+\sigma_V^2). \qquad (5)$$

If σ_P is projected onto the retina, i.e., if D is the distance at which a picture is viewed and l is the focal length of the eye lens (which will be taken as 17.18 mm), then the half-width of the physical spread function projected onto the retina is $(l/D)\sigma_P$. Equation (5) can now be written

$$r=a+b\log\{[(l/D)\sigma_P]^2+\sigma_V^2\}. \qquad (6)$$

It has been proposed by Stevens[17] that an exponential model rather than a logarithmic model should, in general, fit subjective-response data better. An exponential model was tried, but it was found that the fit of the experimental results was not improved. It will be necessary, however, to obtain much more experimental data before a significant choice between models can be made.

III. PHYSICAL EXPERIMENT

In order to obtain experimental data from which values of σ_V and the coefficient b can be obtained, it is essential that each of the pictures which will be judged for definition be generated with a spread function having a constant size and shape. In other words, the same spread function must be used to generate the picture at every point over its area. This is quite

[10] H. J. Zweig, G. C. Higgins, and D. L. MacAdam, J. Opt. Soc. Am. 48, 926 (1958).
[11] E. M. Lowry and J. J. DePalma, J. Opt. Soc. Am. 51, 740 (1961).
[12] J. J. DePalma and E. M. Lowry, J. Opt. Soc. Am. 51, 474 (1961) (abstract only).
[13] R. N. Wolfe, J. Opt. Soc. Am. 49, 1133 (1959).
[14] E. W. H. Selwyn, Phot. J. 88B, 6 (1948).
[15] O. Schade, RCA Review 9, 5 (1948); and later papers.
[16] R. L. Anderson and T. A. Bancroft, *Statistical Theory in Research* (McGraw-Hill Book Company, Inc., New York, 1952), p. 60.
[17] S. S. Stevens, Daedalus 88, 606 (1959); Am. Scientist 48, 226 (1960).

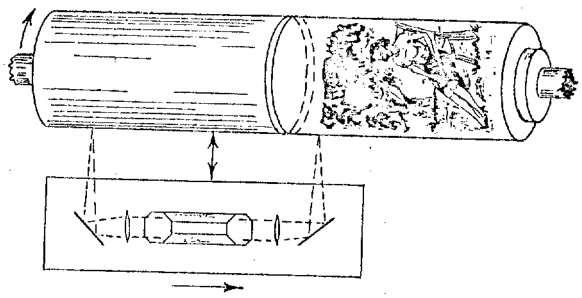

FIG. 1. Diagram of apparatus for "photographing" a transparency so that the spread function is uniform over the entire picture. The transparency is wrapped around the right-hand end of the drum and the negative film around the left-hand end.

different from the situation ordinarily encountered in practical photography. Here a sizable angular field is covered simultaneously, and the spread function which generates the central area of the picture is radically different in size and shape from the spread function which generates the corner regions.

Since ordinary cameras will not produce pictures in which the generating spread function is constant over the field, other means must be adopted to obtain such pictures. A method that has been found effective[8] is shown schematically in Fig. 1. A drum is mounted on a machinist's lathe. The right-hand end is transparent and the transparency to be photographed, which is considered to be the original scene, is wrapped around it. The left-hand end is opaque and the negative film on which the image is to be formed is wrapped around it. On the tool holder of the lathe an optical system consisting of a Dove prism, two 125-mm, $f/5$ lenses, and two mirrors is mounted as shown. The optical system images a portion of the transparency subtending a field of about 0.75° on the negative film. As the drum rotates, the tool holder is translated lengthwise so that the transparency is scanned spirally. The transparency is illuminated from inside the drum with a 75-w, 110-v G.E. photo enlarger lamp operated on direct current at 90 v and held on the optical axis of the system by a mechanical linkage with the tool holder. The operation is similar to cutting a screw in the ordinary use of the lathe, except that there is considerable overlap of the successive tracks of the scan.

The effective spread function of the lens can be changed by varying the distance of the optical system from the drum. Once set, it remains constant over the entire picture.

It is helpful to consider the way in which a letter like P would be transferred by this scanner. If the P were on the positive transparency, it would be transferred through this system onto the negative so that it would have exactly the same orientation. In fact, as the drum is turned, this image would appear as if it were stuck on the drum when it was in the field of the optical system. Every element of the picture is imaged optically in this manner. It should be pointed out that this is different from some photoelectric scanning systems, in which a photoelectric cell convolutes a relatively large number of optical spread functions which are then transferred electronically.

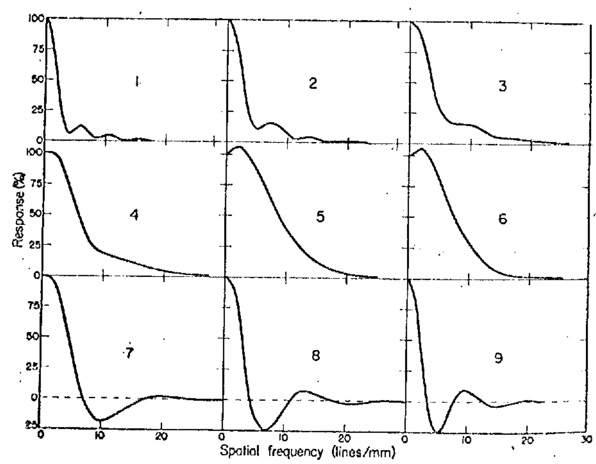

FIG. 3. Frequency-response curves for each of the nine focal settings. The separation between settings was 0.01 in.

A reproduction of this positive transparency used as the "original scene" in this experiment is shown in Fig. 2. Also shown in this figure is a reproduction of a sinusoidal test object which was wrapped around a drum alongside the positive transparency. The test object was thus imaged at precisely the same focal setting as the positive transparency.

In the actual experiment, the lines were oriented at 45° to the lathe axis instead of being parallel to it, as shown in this figure. Experience has shown that this procedure gives results that are consistent with previously reported data[7,8] and also that they show more

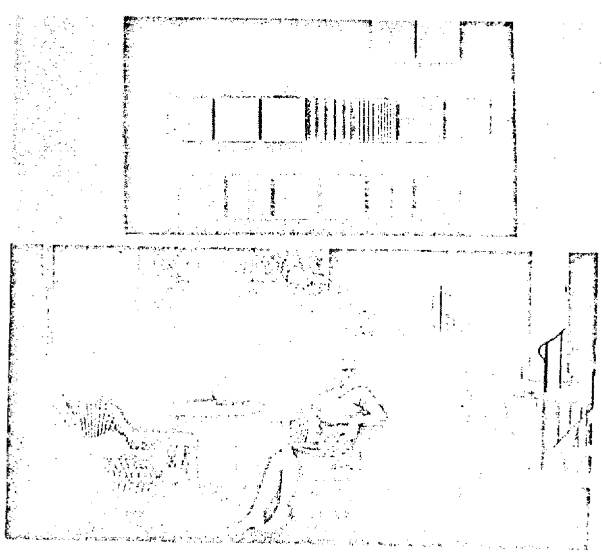

FIG. 2. Test object used in the apparatus shown in Fig. 1. The upper part consists of patterns used for obtaining sine-wave response data and the lower of a pictorial scene.

...mal consistency than do parallel or perpendicular ...

This exposing procedure was used to make nine ...atives. The character of the spread function and ...efore the definition was changed by setting the ...al system at a different distance from the drum ...ween successive negatives. All the negatives were ...e on Kodak Commercial Film and were developed ...ether in a sensitometric processing machine by using ...h agitation to ensure uniformity of development. ...y were developed in Kodak Developer D-76 for 18 ...n at 68°F.

These negatives and the associated sinusoidal charts ...re then carefully printed by contact in a vacuum ...ating frame onto this same kind of film, which was ...veloped in the same manner to provide final positive ...ansparencies for judging and measuring. The sine-wave response[17a] for each positive transparency was ...termined[18] by means of microdensitometer measurements on the photographic images of the sinusoidal ...art made along with it.

TABLE I. Size of physical spread function in terms of standard deviation σ_P on the photograph and for three viewing distances.

Focal setting	Spread-function width $\sigma_P(\mu)$ On photograph	Projected on retina 21 in.	35 in.	56 in.
1	99.07	3.19	1.91	1.20
2	74.73	2.41	1.44	0.90
3	53.11	1.71	1.03	0.64
4	34.06	1.10	0.66	0.41
5	22.48	0.72	0.43	0.27
6	25.17	0.81	0.49	0.30
7	41.42	1.33	0.80	0.50
8	61.22	1.97	1.18	0.74
9	79.70	2.57	1.54	0.96

In this experiment, essentially symmetrical spread functions $f_P(x)$ were used to generate the negatives. These were calculated by taking the Fourier transform of the sine-wave response data for each focal setting. The study of unsymmetrical spread functions is being undertaken, but it was started after the work reported in this paper was completed.

The frequency-response curves derived from the positive transparencies are shown in Fig. 3 for each of the focal settings at which pictures were made. The distance between successive settings was 0.01 in. The

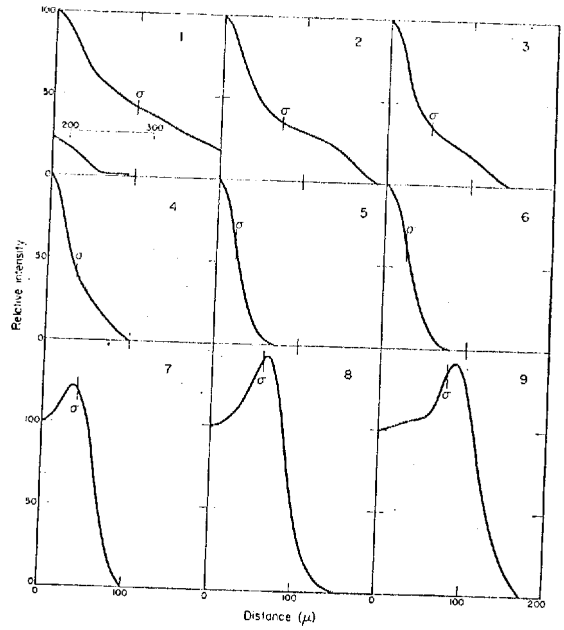

FIG. 4. Spread functions corresponding to the response curves of Fig. 3, with values of standard deviation σ indicated. Since the functions are bilaterally symmetrical, only one half of each is shown.

Fourier transforms of these curves, which are the spread functions, are shown in Fig. 4.

The standard deviation or half-width σ_P is indicated on each curve. These values are shown in Table I and are plotted as a function of focal setting u in Fig. 5. As can be seen, a smooth curve fits these points quite well. This means that some systematically varying property of the physical image is being measured and that random variations are small.

IV. PSYCHOMETRIC EXPERIMENT

Psychometric data were obtained from the nine positive transparencies by two methods. An excellent discussion of these two methods has recently been given

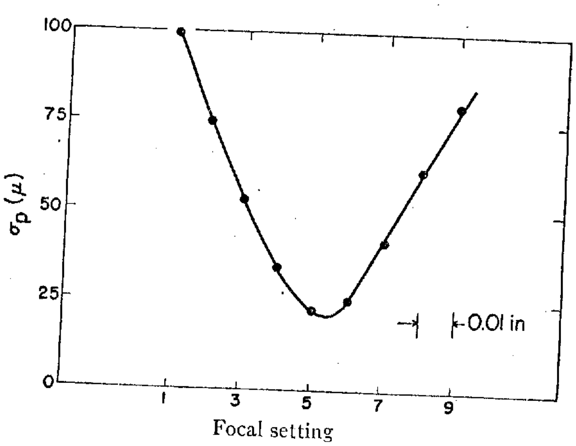

FIG. 5. Standard deviation of spread function as a function of focal setting u.

[17a] *Note added in proof.* A note in the December, 1961, issue of the J. Opt. Soc. Am. 51, 1441, announced an agreement with the ICO Subcommittee for Image Assessment Problems to change "what is at present known under a variety of names, e.g., sine-wave response, frequency response, contrast transfer, etc.," to "optical transfer function" when the complex function (i.e., including phase) is intended, as in Part I of this series. The agreement also provides the term "modulation transfer function" for use when phase is not pertinent. These nomenclature recommendations were adopted too late to be incorporated into this paper, but the new nomenclature will be used in future papers.

[18] R. L. Lamberts, J. Opt. Soc. Am. 49, 425 (1959).

TABLE II. Subjective picture definition $\hat{r}$ as a function of the focal settings at which the photographs were made and the distances from which they were viewed. The method of paired comparisons was used for judging pictures.

Focal setting	Picture definition 21 in.	35 in.	56 in.
1	2.24	5.56	8.17
2	5.26	7.77	9.09
3	6.94	8.77	9.28
4	8.55	9.56	9.74
5	9.34	9.69	9.87
6	9.17	10.41	9.84
7	8.22	9.85	9.57
8	6.45	8.71	9.90
9	4.30	7.31	9.41

by Gulliksen.[19] The first one used was the paired-comparison method, in which all $n(n-1)/2$ or 36 possible combinations of pairs of pictures without regard to order[20] were shown to a number of observers on a diffuse illuminator giving a luminance of approximately 70 ft.-L for the visual field. Each observer was asked to state which of the two pictures i and j in each pair had the better picture definition. The percentage of the total number of observers who considered the picture i to have better definition than picture j was determined for each of the 36 pairs. From these percentage values, the normal variate Δ'_{ij} between each pair was determined. The units in which normal variate is expressed were taken as the units of response (r). The best estimate of the true value of r, which will be designated $\hat{r}$, for each of the transparencies was then computed by Morrissey's procedure[6] mentioned earlier.

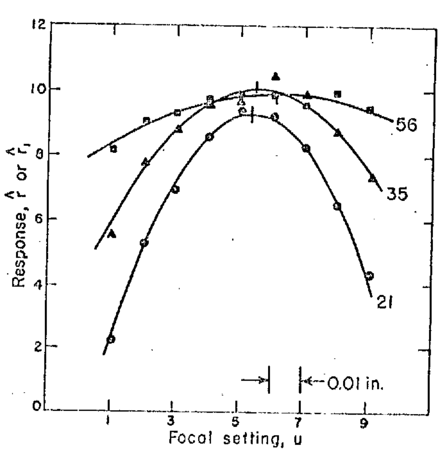

FIG. 6. Average response $\hat{r}$ of 50 observers as a function of focal setting u for the three indicated viewing distances by the paired-comparison method. The curves are quadratic functions fitted to the points and represent $\hat{r}_1$. The maxima are indicated by ticks.

[19] H. Gulliksen, Am. Scientist 47, 178 (1959).

[20] Only one observation of a single pair of pictures, say A and B, was made; the presentation of A on the left and B on the right was considered to be equivalent to the presentation of A on the right and B on the left.

Essentially this procedure is to solve the equations

$$\Delta'_{ij}=r_i-r_j+e_{ij}, \tag{7}$$

where e_{ij} is the observational error. These equations are solved for those values of $\hat{r}$ which satisfy the relation

$$\sum_{\alpha=1}^{n}\hat{r}=0 \tag{8}$$

and for which

$$\sum_{\text{all pairs}} e_{ij}^2,$$

that is, the sum of the squares of the errors, SSE, is a minimum. The restriction imposed by Eq. (8) is made because all the values of $\hat{r}$ are relative to an arbitrary zero.

The transparencies were viewed by 50 observers at each of three viewing distances—21, 35, and 56 in.—and a value of $\hat{r}$ was determined for each transparency at each distance. These values, with the zero selected so that there are no negative values, are shown in Table II and are plotted as a function of focal setting in Fig. 6.

It seems reasonable to assume that $\hat{r}$ should be a smooth function of focal setting u. Quadratic functions of the form

$$\hat{r}_1=\alpha+\beta u+\gamma u^2 \tag{9}$$

were fitted to these data and are shown as the solid curves in the figure. As can be seen, the data points fit the curves quite well. The maxima are indicated on the curves by ticks. The respective focal settings at which the maxima occur are as follows:

Viewing distance (in.)	Focal setting u
21	5.35
35	5.47
56	6.10

The values are plotted in Fig. 7. It should be noted that the focal settings giving maximum response vary

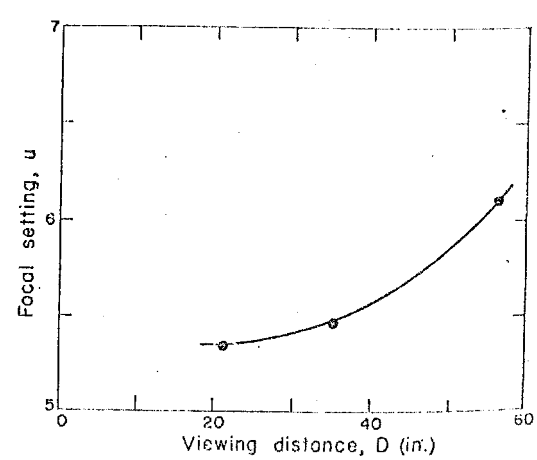

FIG. 7. Variation of focal setting u for maximum of subjective response $\hat{r}_1$ with viewing distance D.

matically with viewing distance. This will be ussed in more detail later.

The second psychometric method was the method of ads." It is illustrated in Fig. 8 by a sketch from liksen.[19] In this method, the pictures are presented groups of three instead of in pairs, and all possible ups of three without regard to order are shown to the ervers. For n pictures there are $n(n-1)(n-2)/2$ h groups. In this case, n is 9, giving 252 groups.

As can be seen from Fig. 8, the pictures are presented a triangular array. The upper picture, at the vertex the array, is taken as the standard to be compared h the two lower pictures. The observer is asked to ide whether the picture at the lower left or the ture at the lower right is *more like* the upper picture. this method it is not necessary to use any verbal ms such as "definition" or "detail" in instructing the ges, as is the case for the paired-comparison method. ntrained observers can thus be used, and, furthermore, the various pictures differ in more than one respect, if the psychometric domain is multidimensional, the nal results will indicate the number of different attributes in which the pictures vary. They will not, however, indicate the *nature* of the different attributes. That can be determined only by subsequent study.

These judgment data are first used to measure the interpoint distances between pictures.[19] These interpoint distances are determined in a manner analogous to the way in which the response values for each picture are determined in the paired-comparison method. The number of interpoint distances is obviously equal to the total number of pairs of pictures—36 in the case of 9 pictures.

These interpoint distances are absolute values and contain no information as to direction. In other words, given three interpoint distances, AB, BC, and AC, AC is not necessarily equal to $AB+BC$. The points A, B, and C might lie along a line, but alternatively, they might lie on a plane in two-dimensional space or, in the case of a larger number of points, even in some space of higher dimensions. Statistical techniques[21–23] have been developed to fit these various points into one-, two-, three-, or n-dimensional space and to determine how many dimensions are needed to account adequately for the various interpoint distances. Thus, if it can be shown that the points obtained from picture judgments can be fitted more closely by a plane than by a line and that the difference in degree of fit indicated by statistical tests is significant, then the pictures probably do not differ in just one subjective attribute but in two.

Data using this triad technique were obtained for the same transparencies that were evaluated by the paired-comparison technique. Two separate groups of 30 observers were used. The first group consisted of

TABLE III. Subjective picture definition $\hat{r}$ as a function of the focal settings at which the photographs were made for a viewing distance of 21 in. The method of triads was used for judging pictures.

Focal setting	Picture definition Trained obs.	Untrained obs.
1		7.65
2		8.70
3	8.32	9.16
4	9.15	9.62
5	9.67	9.93
6		9.80
7	8.77	9.68
8	7.62	9.18
9		8.36

laboratory personnel who had a knowledge of the nature of the experiment and who had made observations of the transparencies by the paired-comparison technique. These will be called "trained" observers in our discussion. Because of the magnitude of the triad program as compared with the paired-comparison program, the trained observers were asked to judge only five of the nine transparencies.

The second group, which will be called "untrained" observers, consisted of female personnel from the Motion Picture and Sheet Film Division, whose primary duties are to inspect film under a low level of safelight illumination. Their visual acuity was definitely better than average. Moreover, they were all interested in the observing program, which was somewhat different from their usual duties, and they were paid at their customary rate. Consequently, they seemed to be adequately motivated. In both these cases, the data could be fitted adequately to a line, indicating that the observers had judged the pictures on the basis of only a single psychometric attribute.

Table III shows the response values so obtained for the separate groups of observers. All these values were obtained for a viewing distance of approximately 21 in. The attribute which was observed seems to be definition. These values, designated also by $\hat{r}$, are plotted as func-

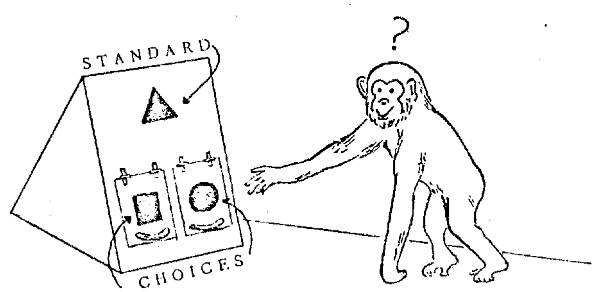

FIG. 8. Illustration of method of triads. The perplexed simian is trying to decide whether the square or the circle is more nearly like the triangle. (From Gulliksen, by permission. © *American Scientist.*)

[21] G. Young and A. S. Householder, Psychometrika 3, 19 (1938).
[22] S. J. Messick and R. P. Abelson, Psychometrika 21, 1 (1956).
[23] S. J. Messick, Ph.D. thesis, Princeton University, and Educational Testing Service, Princeton, New Jersey (1954).

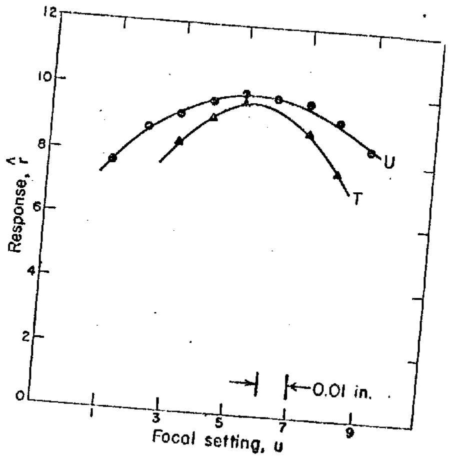

FIG. 9. Average response $\hat{r}$ of 30 observers as a function of focal setting u for a viewing distance of 21 in. by the method of triads. U, untrained observers; T, trained observers.

tions of focal setting in Fig. 9 as circles for the untrained observers and as triangles for the trained observers.

Analysis of Data

Both the triad and the paired-comparison response data are adjusted so that a picture made with a "perfect" physical system, i.e., a system in which $\sigma_P=0$, will have a response of 10. This, of course, requires a knowledge of the values of the parameters σ_V and b in Eq. (6), which have not been given yet in this paper. Since the *zero* on the various response scales is arbitrary, this procedure is justified in order to avoid showing two sets of response curves which are essentially duplicates.

The values of b and σ_V can be obtained by starting with Eq. (6). This equation, the proposed mathematical model, can be written as

$$\begin{aligned} r&=f(\sigma_P, D; a, b, \sigma_V)\\ &=f, \end{aligned} \tag{10}$$

and for a particular stimulus, i.e., for the case when a particular picture is viewed from a particular distance,

$$r_i=f_i.$$

The procedure for estimating values of b and σ_V is analogous to that used for obtaining values of $\hat{r}$ from Eq. (7). Since r is relative to an arbitrary zero, the value of a is arbitrary. As in Eq. (7), Δ'_{ij} is the normal variate between each pair of pictures judged at a particular viewing distance or an analogous quantity obtained by the method of triads. Following the previous procedure, we solve the equations

$$\Delta'_{ij}=(f_i-f_j)+e_{ij} \tag{11}$$

for those values of b and σ_V which make SSE a minimum, i.e., for which

$$\sum_{\text{all pairs}} e_{ij}^2$$

is as small as possible. The estimates of the true values of b and σ_V obtained in this way will be symbolized by $\tilde{b}$ and $\tilde{\sigma}_V$. Likewise, the estimate of r obtained by substituting $\tilde{b}$ and $\tilde{\sigma}_V$ into Eq. (6) will be symbolized by $\tilde{r}$.

With $\tilde{b}$ and $\tilde{\sigma}_V$ found, it would be of interest to know how much they might vary and still be in reasonable accord with the data. This can be determined by applying significance tests such as are outlined by Brownlee.[24]

Two such tests were made. One was made for the combination of all the data from the paired-comparison evaluation at the three viewing distances. The other was made for the triad evaluation by the untrained observers. In the case of the trained observers using the triad method, there were insufficient data to make a useful test because only five of the nine pictures were evaluated.

The results of these tests are shown in Fig. 10. The closed loop P is for the paired-comparison data obtained by the trained observers, and the open figure T is for the triad data obtained by the untrained observers. All pairs of b and σ_V values inside the loop and between the sides of the open figure are jointly compatible with the data at or better than the 5% level of significance. In other words, we have 95% confidence that the true[25] values of b and the corresponding values of σ_V will lie within these figures.

For the paired-comparison data, the SSE (sum of the squares of the errors) of Eq. (11) has a minimum at A'. On the boundary of the figure it has a slightly higher but constant value, which means that all points

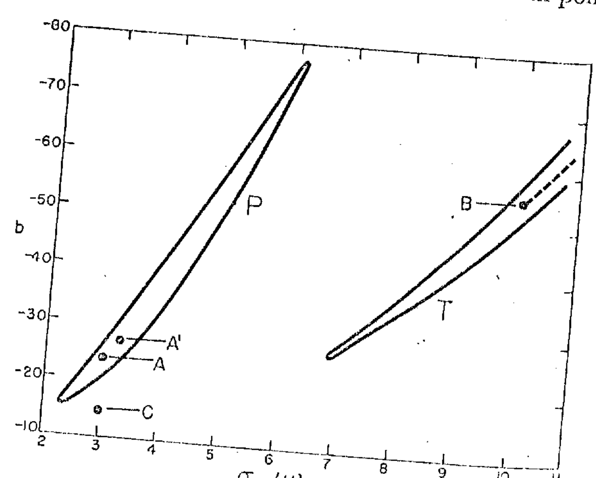

FIG. 10. Value of b as a function of σ_V in Eq. (6) from (P) paired-comparison data for all viewing distances and (T) triad data for a 21-in. viewing distance. The significance of points A, A', B, and C is explained in the text.

[24] K. A. Brownlee, *Statistical Theory and Methodology in Science and Engineering* (John Wiley & Sons, New York, 1960), pp. 303–306.

[25] "True" is here used to mean those values of b and σ_V which would be obtained if there were no random errors in the experiment.

on this boundary are equally consistent with the data. For the triad data, the SSE is lower along the dotted line through B to the right than elsewhere on the figure and is slightly higher but constant along the boundary, where again all points are equally consistent with the data.

Although there were inadequate data to make a useful significance test for the triad data of the trained observers, a very shallow minimum SSE was obtained that led to point C, where σ_V is 3 μ and b is -14.8. These values of $\tilde{\sigma}_V$ and $\tilde{b}$ were used for calculating the corresponding curve (through the triangles) in Fig. 9. Likewise, the values obtained from point B in Fig. 10, where $\tilde{\sigma}_V$ is 10μ and $\tilde{b}$ is -55.8, were used for calculating the curve through the circles in Fig. 9.

Values obtained from point A instead of point A' were used for calculating the curves of $\tilde{r}$ vs focal setting for the cases where the trained observers used the paired-comparison method. To determine point A, the value of σ_V at A' was simply rounded off to 3 μ from 3.25 μ, thus making it the same as for C. Since A is within the figure, it is, like A', consistent at better than 95% level of significance. The value of $\tilde{b}$ associated with $\sigma_V = 3$ μ is -23.6. The curves of $\tilde{r}$ vs focal setting corresponding to A are shown in Fig. 11.

Before proceeding to discuss the curves in Figs. 9 and 11, it should be pointed out that the data as analyzed in Fig. 10 strongly suggest that σ_V for the untrained observers using the triad technique is larger than σ_V for the trained observers using the paired-comparison technique, but the values for b are probably the same in both cases. Since significant tests could not be made for the data obtained by the trained observers using the triad technique, we cannot say whether this difference indicates differences between observers or is an artifact caused by the differences in technique. Further experiments are needed to clear up this point.

As can be seen in Fig. 9, the untrained observers could scarcely tell the picture which is highest on their response scale from a picture made with a "perfect" physical system, while the trained observers could definitely detect this difference. Also the untrained observers report relatively higher response values than the trained observers for the out-of-focus pictures, indicating that the untrained observers do not detect differences in the pictures that are apparent to the trained observers. Nevertheless, regardless of these qualitative observer variations, this mathematical model seems to give an adequate explanation for the experimental results obtained at a single viewing distance.

Now let us see what response values the mathematical model predicts in the case of the data obtained at the three viewing distances.

Figure 11 shows the curves of $\tilde{r}$ adjusted so that a "perfect" optical system would give a response value of 10. The points $\hat{r}$ calculated by Morrissey's method[6] are also shown in the figure. As can be seen, the model

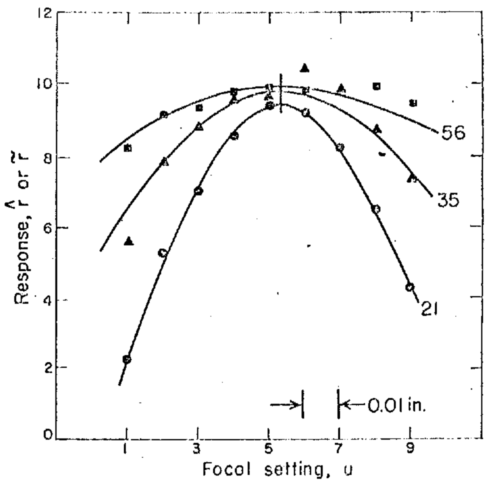

FIG. 11. Average response $\hat{r}$ of 50 observers as a function of focal setting u for the three indicated viewing distances (in inches) by the paired-comparison method. The curves are functions that minimize the SSE and represent $\tilde{r}$. The maxima are indicated by ticks.

agrees with the experimental points quite well. It predicts maximum responses of 9.4, 9.8, and 9.9 for the 21-, 35-, and 56-in. viewing distances, respectively. Moreover, the variation in definition as a function of focal setting gets progressively less pronounced as the viewing distance is increased.

These results probably mean that, at the 56-in. viewing distance, these trained observers do not distinguish the picture having the best definition from a picture with "perfect" definition.

The experimental points in Fig. 11 show a tendency, not predicted by the model, for the curves representing the model to shift slightly to the right with increasing viewing distance. Such a shift was mentioned earlier and was demonstrated in Fig. 6. It was found that the data for each viewing distance could be fitted to a quadratic function, Eq. (9), and that no significant improvement in fit could be obtained by using a function higher in degree than a quadratic. In addition, calculation shows that the shift of the focal setting with viewing distance for the calculated maxima is significant at the 95% level.

This systematic deviation of the values of $\hat{r}_1$, representing the experimental results, from the values of $\tilde{r}$, computed from the model, can be seen more easily by replotting the data from Fig. 11 as in Fig. 12. In the left-hand graph, the response values for the 21-in. viewing distance are plotted as ordinates and those for 35 in. as abscissas. In the right-hand graph, the values for the 35-in. viewing distance are plotted as ordinates and those for 56 in. as abscissas. That is, the shorter viewing distance is represented by ordinates in both figures.

The best experimentally determined values of $\hat{r}$ are indicated by the small circles that are numbered to indicate the focal settings at which they were made.

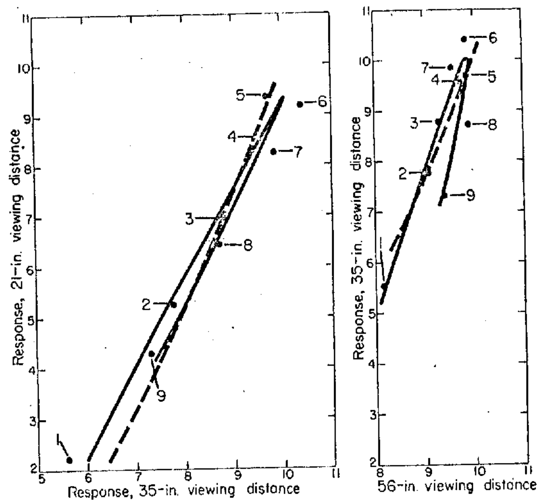

FIG. 12. Subjective response for relatively short viewing distances as functions of response for relatively long distances. Circles, experimental points $\hat{r}$, numbered according to the focal settings at which they were obtained; solid curves, values $\hat{r}_1$ from best quadratic function for each viewing distance (Fig. 6); broken curves, values $\bar{r}$ from mathematical model (Fig. 11).

By following these points in numerical order, a definite, if rough, clockwise loop structure is evident in both graphs. The values $\hat{r}_1$ from the quadratic formula, Eq. (9), are represented by the solid curves. These curves also show definite loop structures. The values $\bar{r}$ given by the mathematical model, Eq. (6), are indicatde by the broken curves. These curves are, of course, monotonic.

Inspection of the figure shows that this mathematical model predicts quite closely the picture definition of pictures that are generated by spread functions of different shapes and sizes and viewed at different distances but that, at least in this experiment, there is a significant systematic deviation between the predicted and the observed values. This deviation may be related to the fact that the spread functions generating the pictures on the left-hand sides of the loops, pictures 1–4, have a single central peak while those on the right-hand side of the loops, pictures 7–9, tend to be bimodal. This change in shape of the spread function is a typical manifestation of spherical aberration in an optical system.

V. SIZE OF VISUAL SPREAD FUNCTION

It is of interest to compare the results of this experiment with those obtained by other workers in this general field. A summary of the various data is given in Table IV. In the present experiment a value of σ between 3 and 6 μ for the visual system was obtained by using the paired-comparison method and a value greater than 8 μ was obtained by using the triad method. The viewing-field luminance was approximately 70 ft.-L.

A value for σ_V was derived from data presented by Selwyn[14] in 1948. He suggests that his visual resolving-power data can be explained by "supposing that the retina averages the illumination over a band width of 1.25 min." Assuming this width is the 2σ value and using 17.18 mm for the focal length of the eye, as was used earlier in this paper, we have a σ value for the retinal spread function of 3.1μ.

If the pupil diameter is taken as 3.0 mm, the σ value for the diffraction-limited image[9] is 1.4 μ. The square root of the sum of the squares of these two values gives σ_V as 3.4 μ.

A value of 9 μ was derived from data presented by Schade[15] in 1948 for a field luminance of 7 ft-L and a value of 5 μ was derived for a field luminance of 70 ft-L from data which he presented in 1956.[26] In each case he presented his observers with a series of bar resolving-power charts varying in line spacing and required that the ratio of line-to-surround luminance be adjusted until threshold conditions were obtained, i.e., until the pattern at each of the spacings could just be seen.

Ludvigh[27] in 1953 assembled data from various sources for the size of the "blur disks" for diffraction, chromatic and spherical aberrations, fixation, tremor, and uncorrected refractive error. By adding the diameters of these disks together, he obtained an over-all value of 754 sec of arc, or a diameter of 63 μ on the retina, for an eye with a 4-mm pupil. This pupillary diameter is obtained with a 50-ft-L field luminance. Instead of a simple sum, a root-mean-square value is probably preferable, which reduces this diameter to 40.5 μ. If we assume Fry's[28] relationship between σ and the diameter of the blur circle Φ, i.e.,

$$\sigma=\Phi/(2\pi)^{\frac{1}{2}},$$

then $\sigma=16$ μ.

A value of 10 μ was derived from data obtained by Flamant[29] in 1955. She photometered retinal images with a physical instrument, which means that the physiological-psychological elements of the retinal system were not included. No meaningful figure for field luminance could be given for such an experiment.

A value of 14 μ was derived from observations of photographic graininess made by Stultz and Zweig[30] in 1959. They determined by least-squares methods the size of retinal image which would best fit their data. This was done by using a method similar to that used in the present experiment. Their viewing-field luminance was less than 10 ft-L as reported to the author in a private communication.

Lowry and DePalma[12] have just reported data indicating that σ in their case was less than 3 μ. Their data, which were derived from observations of the Mach

[26] O. Schade, J. Opt. Soc. Am. 46, 721 (1956).
[27] E. Ludvigh, U. S. Naval School of Aviation Medicine and Kresge Eye Institute, Bureau of Medicine and Surgery, Project No. NM 001 075.01.04, 17 August, 1953.
[28] G. A. Fry, *Blur of the Retinal Image* (Ohio State University Press, Columbus, Ohio, 1955).
[29] F. Flamant, Rev. optique 34, 433 (1955).
[30] K. F. Stultz and H. J. Zweig, J. Opt. Soc. Am. 49, 693 (1959).

TABLE IV. Summary of determinations of size of the human visual spread function. References are to footnotes in the text.

Author(s)	Date[a]	Technique	Field luminance (ft-L)	Half-width, σ_V (μ)
Wolfe	1962	Picture viewing; paired comparison	70	3–6
Wolfe	1962	Picture viewing; triads	70	>8
Selwyn[b]	1948	Threshold		3.4
Schade[c]	1948	Threshold	7	9
Schade[d]	1956	Threshold	70	5
Ludvigh[e]	1953	Composite blur disk	50	16
Flamant[f]	1955	Physical photometry		10
Stultz and Zweig[g]	1959	Graininess observations	<10	14
Lowry and DePalma[h]	1961[i]	Mach phenomenon	20	<3

[a] Unless otherwise indicated, date of archival publication.
[b] See reference 14.
[c] See reference 15.
[d] See reference 26.
[e] See reference 27.
[f] See reference 29.
[g] See reference 30.
[h] See reference 12.
[i] Date of oral presentation.

phenomenon, were obtained at a field luminance of 20 ft-L.

As can be seen, the results of the present experiment are consistent with those obtained by other workers. Moreover, they relate to actual picture observations and include both the objective and subjective elements of the human visual system.

VI. CONCLUSION

We may conclude that the mathematical model proposed here predicts quite closely the picture definition to be seen in pictures generated by spread functions of different shapes and sizes and viewed at different distances, and that the half-width σ of the spread function for the human visual system is between 3 and 8 μ. There are, however, small systematic differences between the experimental data and the values predicted by the model. These differences are probably related to gross differences in shape among the various spread functions.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance given by Sam A. Tuccio and Bryce E. Bayer of these Laboratories. He is indebted to Mr. Tuccio for assisting him in the experimental work and making the many detailed mathematical calculations and to Mr. Bayer for advice and consultation about the theory used for determining the statistical significance of the data.

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THE USE AND CALIBRATION OF THE 'MAXWELLIAN VIEW' IN VISUAL INSTRUMENTATION

The radiant energy reflected by or emitted from a surface diverges in all directions.[1] In order to be effective visually, this energy must pass through the entrance pupil of the eye after which, assuming adequate dioptrics, it will be focused on the retina. The size of the light pencil emanating from a given object point which reaches the corresponding image point on the retina is proportional to the pupil diameter. The larger the pupil diameter (or the light pencil) the more energy from the object point will be able to reach its corresponding image point and the 'brighter' the point will appear. The entrance pupil of the eye can, however, intercept only a small fraction of the energy from a diffuse surface, and consequently most of the emitted energy does not enter the eye and is therefore not effective visually (Fig. 1A).

In order to present an extended test-field of high luminance, the 'Maxwellian view'

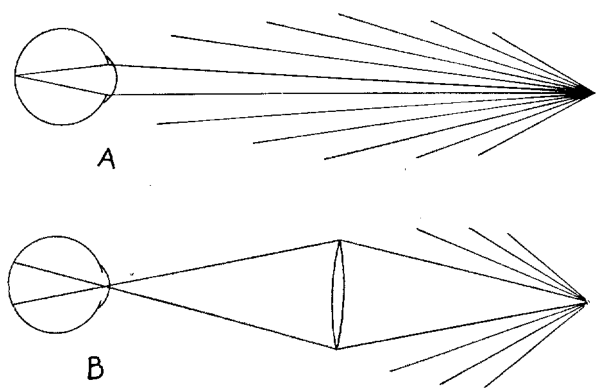

FIG. 1. DIAGRAM ILLUSTRATING THE EFFICENCY OF THE 'MAXWELLIAN VIEW'
A, unaided eye, only the light intercepted by the pupil reaches the retina; B, Maxwellian view, all the light intercepted by the lens enters the eye.

is frequently used in visual instrumentation.[2] The principle of the Maxwellian view is to focus the image of a light source in the plane of *S*'s pupil (see Fig. 1B). All the light intercepted by the lens enters the eye as contrasted with Fig. 1A where

[1] The author wishes to express his appreciation to Dr. C. S. Bridgman for his effective aid in the preparation of this note.

[2] Named in honor of J. Clerk Maxwell who utilized this principle in the construction of his color-mixing apparatus. (See Maxwell, On the theory of compound colors and the relation of the colors of the spectrum, *Phil. Trans.*, 150, 1860, 57.)

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forms a conjugate image of the source with unit magnification at some distance away from the eyepiece of the apparatus). Alternative procedures for the calibration of the Maxwellian view which permit the photometer to be used on diffuse surfaces are indicated below.

A convenient technique for calibration of the Maxwellian view is to cover one half of the field lens with a mirror positioned so as to reflect the image of a diffuse surface in the direction of *S*'s eye. *S* then sees a bipartite field, one half of which is supplied by the Maxwellian view field lens and the other half by the image of the diffuse surface seen in the mirror. The two halves of the field lens are matched by adjusting the luminance of either component after which a calibration value is obtained in the usual manner from the diffuse surface. As in the calibration method discussed above, the effective entrance pupil must be the same size for the Maxwellian and the diffuse fields.[6]

In the binocular method *S* observes the field of the Maxwellian view with one eye and a diffuse surface of the same size with the other eye. By adjusting convergence, the two fields may be made to appear in juxtaposition. After being matched for equality, the luminance is determined with a conventional photometer from the diffuse surface. It is necessary, of course, that the entrance pupils of the two eyes be equal. The facility with which this binocular match can be made is improved by providing for a common boundary between the two fields. This can be accomplished by presenting both fields in the form of a semi-circle or rectangle. The variability of binocular matches is, however, greater than for monocular comparison.

A third technique makes use of a photo-electric circuit.[7] The sensitive surface of the photocell is placed in the plane normally occupied by *S*'s pupil and the voltage generated by the image focused there is recorded. A diffuse surface equal in size to the field lens is illuminated and the light reflected into the photocell located at a distance equal to that between the eye and the field lens. The area of the photocell exposed to this light must be equated with the area of the image of the source previously focused on the photocell. The illumination on the diffuse surface is varied until the flux density of the light reflected on the photocell surface, as indicated by the voltage generated there is equal to the flux density previously produced there by the Maxwellian view system. Under this condition the effective energy entering the *S*'s pupil in the Maxwellian system is equal to that which would enter the pupil from the diffuse source. The luminance of the diffuse source can then be determined in the conventional manner.

Summary. The principle of Maxwellian view is advantageously used in visual experimentation when it is desirable to obtain extended visual stimuli of high luminance. Precautions in the use of this system as well as some advantages and disadvantages are discussed. Calibration of the system involves special problems when using conventional photometers. Alternative methods for calibration which make use of monocular comparison, binocular comparison, and a photoelectric circuit are described.

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[6] The author is indebted to Dr. Yun Hsia for the details of this technique.
[7] Described by K. J. W. Craik, The effect of adaptation on differential brightness discrimination, *J. Physiol.*, 92, 1938, 406.

only the light intercepted by the pupil enters the eye. Subjectively, *S* sees the lens (called the field lens) filled with light. With the aid of additional lenses the light can be rendered parallel hence, neglecting absorption in the lens and surface reflections, it can transverse long optical paths without loss. Variations in temporal characteristics, direction, and energy may be produced by modification of this beam by the appropriate shutters, lenses, reflecting surfaces, filters, etc.

An advantage of the Maxwellian view, in addition to its efficiency in transmitting energy from the source to *S*'s eye and the possibilities it offers for manipulation and control of the light, is the elimination of fluctuations in natural pupil size as an experimental variable provided the image of the source in the plane of the pupil is smaller than *S*'s natural pupil. In addition, since light from each point of the source passes through all zones of the lens, irregularities in the source will not affect the uniform appearance of the field lens.[3] It is not advisable, however, for the source to exhibit gross differences in luminances such as would be obtained from a coiled tungsten filament unless the image of the filament is so small as not to be imaged on the edge of the pupil. If the latter situation exists, movements of the eye can result in variations in the total flux reaching the retina as the pupil edge moves across areas of non-uniform flux-density. A disadvantage of the Maxwellian view is the necessity for placing the eye in such a position that the image of the source is centered in the entrance pupil, although this arrangement does permit the presentation of peripheral stimuli without decrease in the effective area of the entrance pupil.[4] Another disadvantage may arise in experiments on visual acuity where the possible additional complication of Abbe diffraction may be introduced by the Maxwellian view optics.[5]

Since most visual photometers (*e.g.* the MacBeth illuminometer) are designed for the calibration of diffuse surfaces, special precautions must be observed if they are to be used in the calibration of Maxwellian view systems. It is necessary, that the appearance of the field lens being calibrated is not changed, that the exit pupil of the photometer and the image of the source in the Maxwellian system be concentric and in the same plane so that the position of the eye is the same for calibration as in the normal use of the system. The aperture-stop of the photometer must also be equal to or smaller than the image of the source in the Maxwellian system that the Maxwellian system and the photometer comparison surface be viewed through the same diameter entrance pupil. This requirement is necessary to satisfy the requirement that specification of luminance be independent of viewing conditions. It is not always possible to satisfy these conditions as the eyepiece of the apparatus may be so constructed as to prevent the proper positioning of the photometer or the aperture-stop of the photometer may be too large. In some cases it may be possible temporarily to alter the design of the apparatus or the photometer so as to satisfy these requirements (*e.g.* by the use of a supplementary lens which

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[3] If the image of the source in the plane of the pupil is too small, entoptic phenomena will destroy the even appearance of the field lens. See A. A. Michelson, On the effect of small particles in the vitreous humor, *J. Opt. Soc. Amer.*, 9, 1924, 197.

[4] See the apparatus used by R. T. Brooke, Variations of critical fusion frequency at various retinal locations, *J. Opt. Soc. Amer.*, 41, 1951, 1010.

[5] *E.g.* the apparatus designed by S. Shlaer, The relations between visual acuity and illumination, *J. Gen. Physiol.*, 21, 1937, 165.

Fundamental Variables in the Relationship Between Accommodation and Convergence*

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Abstract

The present paper is an attempt to demonstrate the usefulness of a graphical analysis of the data obtained in a routine examination that pertain to accommodation and convergence. The relationships between accommodations and convergence are demonstrated to involve five fundamental variables. Once the findings are analyzed in terms of these variables, the reason for the anomalies in the relationship between accommodation and convergence can be more fully appreciated.

Furthermore, the type of graphical analysis which is recommended makes it possible to check findings obtained in a routine examination against each other to determine what findings involve experimental errors.

Maddox's Classification of the Different Forms of Convergence. In a survey of the literature, which has by no means been complete, Maddox[1] appears to have been the first to split convergence into different components. He analyzed convergence into four components as follows:

(1) Tonic convergence.
(2) Accommodative convergence.
(3) Positive fusional convergence.
(4) Negative fusional convergence.

Graphical Analysis of the Relationships Between Accommodation and Convergence. Maddox's analysis of the different components of convergence and their relationship to accommodation can best be illustrated by plotting part of the routine findings in an optometric examination on a graph as illustrated in Fig. 1. It should be noted that this graph is similar to the ones advocated by Sheard[2] and Lesser[3] for routine use in an optometric practice. The following findings can be plotted on the graph.

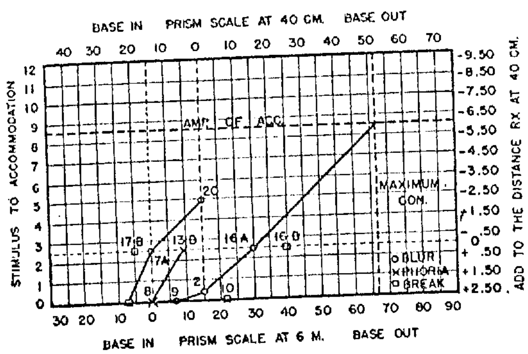

Fig. 1

I. Tests at 6 m. with the subjective.
   (a) Lateral phoria.
   (b) Base in to break.
   (c) Base out to blur and break.
II. Tests at 40 cm. with the anticipated near point correction.
   (a) Lateral phoria.
   (b) Base in to blur and break.
   (c) Base out to blur and break.
   (d) Plus lens to blur.
   (e) Minus lens to blur.
III. Near point of convergence. (Push up method.)
IV. Monocular amplitude of accommodation. (Push up method.)

Construction of the Graph. In constructing the graph it is assumed that a target at 6 m. seen through the subjective finding represents a zero stimulus to accommoda-

*Reprinted from the March 18 and 25, 1943 issues of *The Optometric Weekly.*

[1] E. E. Maddox, *The Clinical Use of Prisms*, pp. 158-176 (1907).

[2] C. Sheard, "Zones of Ocular Comfort," *Trans. of the Amer. Acad. of Optom.*, Vol. 3, pp. 113-129 (1928).

[3] S. K. Lesser, *Fundamentals of Procedure and Analysis in Optometric Examination*, pp. 109-111 (1934).

tion and a target at 40 cm. a 2.50 D. stimulus to accommodation. The value of 2.50 D. is computed by taking the reciprocal of .40 m. Actually some allowance should be made for the fact that 6 m. is used instead of infinity as a basis for computing the zero stimulus to accommodation. If this were taken into account, the stimulus at 40 cm. would be 2.33 D. instead of 2.50 D. This discrepancy can be disregarded in constructing the graph because it makes it easier to represent on the graph the changes in the stimulus to accommodation produced by adding plus or minus lenses to the subjective correction. The various additions with the target at 40 cm. are represented on the right hand side of the graph. If it were necessary to assume that a target at 40 cm. represents a 2.33 D stimulus to accomodation, the divisions on the scale on the right hand side of the graph would not correspond to the divisions on the left hand side.

The stimulus to convergence is assumed to be 15△* greater when the target is at 40 cm. than when the target is at 6 m. Hence the zero point on the prism scale at 40 cm. at the top of the graph is displaced 15△ to the right from the zero on the prism scale at 6 m. on the bottom of the graph. This value of 15△ is only correct for an interpupillary distance of 68 mm. and varies from 12△ to 16△ for interpupillary distances ranging from 55 to 75 mm. These variations can be ignored in constructing the graph. The value 15△ is chosen because it is a round number which makes it possible to use the same vertical lines both for the scale at the top and also for the scale at the bottom of the graph.

The stimulus to convergence can be changed by adding prism base in or base out at distance or near and these stimuli can be located directly by means of the scales at the top and the bottom of the graph. It should be noted that the prism scales are calibrated in centrads instead of prism diopters. The rotary prisms now widely used in practice are calibrated in prism diopters. For the purpose of case analysis, it is satisfactory to regard the prism findings expressed in terms of prism diopters as equivalent to centrads. Once this is done, the various findings can be added together or subtracted one from the other without involving any error.

*Because of the inability of our printer to reproduce the symbol for centrad, we have used the symbol for prism diopter. Throughout this article all prism symbols indicate centrads—Ed.

Procedure in Plotting the Findings. The phoria findings are indicated by crosses, the base in and base out blur points by circles and the break points by squares. Since the phoria and duction findings at 6 m. are made through the subjective finding, the stimulus to accommodation is zero and the findings must be plotted on the horizontal line corresponding to zero on the accommodation scale. The prism scale at the bottom of the graph is used in plotting these findings.

The phoria and duction findings at near when taken through the subjective involve a 2.50 D stimulus to accommodation and are plotted on the 2.50 D horizontal line. The prism scale at the top of the graph is used in plotting these findings.

When the phoria and duction findings at 40 cm. are made through some add to the subjective they must be plotted on the horizontal line corresponding to the add as shown directly on the right hand side of the graph. This procedure is illustrated in Fig. 2.

The plus lens to blur and the minus lens to blur findings are made with the target

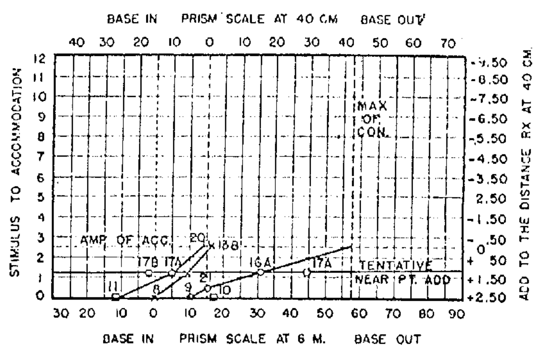

Fig. 2

placed at 40 cm. and since the stimulus to convergence is kept fixed, both findings must be plotted on the vertical line corresponding to the zero point on the prism scale at 40 cm. These findings are represented by circles.

The absolute maximum of convergence as determined by the push up method can be represented on the same graph. In the test for the absolute maximum of convergence, the target is moved toward the eyes until it is seen double or until one eye is seen by the examiner to turn out, and the distance of the near point of convergence from the plane of the spectacles is recorded in centimeters.

The absolute maximum of convergence is measured in terms of the angle of convergence at the near point. This angle can be calculated in terms of degrees by the following formula which is based upon the diagram in Fig. 3.

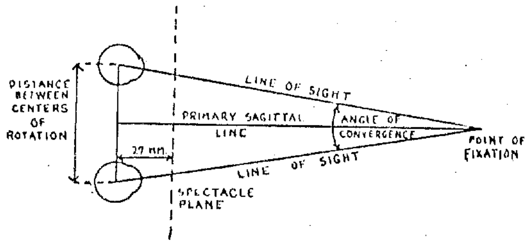

Fig. 3

$$\text{Tan } \tfrac{1}{2} \left[\begin{array}{c} \text{Angle of} \\ \text{convergence} \end{array} \right] = \frac{\tfrac{1}{2} \text{ Interpupillary distance}}{\left[\begin{array}{c} \text{Distance from spectacle plane to} \\ \text{the near point of convergence} \end{array} \right] + .027}$$

The value .027 represents the distance in meters from the spectacle plane to the base line connecting centers of rotation. The angle of convergence measured in degrees can be translated into centrads by multiplying by 1.75. The graph in Fig. 4 eliminates the need of this calculation. The absolute maximum of convergence can then be represented on the graph by erecting a vertical dotted line at a point on the prism scale at 6 m. corresponding to the centrad value of the maximum convergence. No allowance needs to be made for the fact that the zero on the prism scale at 6 m. represents $1\triangle$ of convergence.

The amplitude of accommodation can be indicated by a horizontal dotted line at a point on the stimulus to accommodation scale corresponding to the amplitude. In case the amplitudes for the separate eyes are found to be different this can usually be regarded as an experimental artifact and the higher of the two findings is regarded as the more representative of the amplitude. If the dynamic skiascope finding and the unfused cross cylinder finding also indicate that the accommodative responses are unequal for the two eyes, so that different adds for the two eyes are justified for near point work, the plus lens and minus lens to blur findings have to be disregarded and the prism findings at 40 cm. are plotted in terms of the accommodative response for the eye having the large amplitude.

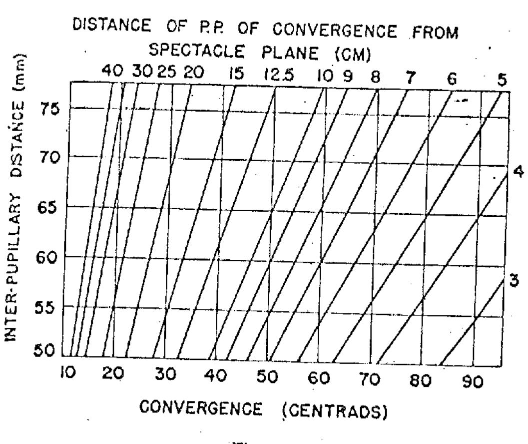

Fig. 4

Significance of the Phoria Finding at Distance. The phoria at distance represents the position which the eyes assume under the influence of the basic tonicity of the extra-ocular muscles. Awareness of nearness is eliminated from the situation by using a target actually placed at 6 m. The stimulus to accommodation is eliminated by using the most plus or least minus lens power before the eye which will permit clear vision. The fusional stimulus is eliminated by dissociating the two eyes.

Significance of the Phoria Finding at 40 Cm. In order to get a measure of the effect of changing accommodation on convergence the eyes can be kept dissociated, the target placed at 40 cm. and a new phoria measurement made at this distance. Changing the distance of the target increases the stimulus to accommodation 2.50 D.D. unless an add is used for making the phoria finding at near. The amount of convergence associated with this change in accommodation can be computed by the following formula: Accommodative convergence = $15\triangle$ — phoria at distance + phoria at near. In applying the formula an eso. is regarded as plus and an exo. as minus. The $15\triangle$ represents the convergence required for binocular fixation of the near point target.

The accommodative convergence associated with each diopter of accommodation will be referred to as the A.C.A. ratio, and can be computed by the following formula:

$$\text{A.C.A. ratio} = \frac{15\triangle - \text{phoria at distance} + \text{phoria at near}}{2.50 - \text{add through which the phoria at 40 cm. is made.}}$$

This ratio is equal to the reciprocal of the slope of the line connecting the phoria findings at distance and near. One can get an approximate idea of the ratio by drawing this line on the graph and making a casual inspection of its slope.

Maddox pointed out that part of this so-called accommodative convergence may be attributable to the awareness of nearness. It is not known whether this involves a psychic stimulation of positive fusional convergence or some unique form of convergence. At least for the time being it is probably better to avoid splitting up the so-called accommodative convergence into two separate components as has been done by Tait[4] and Farmer[5].

Significance of the Prism Blur and Break Findings. Maddox and many of his contemporaries failed to grasp the true significance of base in and base out blur points. As early as 1892 Percival[6] pointed out that the blurring which occurred with prism base in was brought about by a forced relaxation of accommodation and that the blurring resulting from prism base out could be attributed to a forced increase in accommodation. This can be demonstrated by using plus and minus lenses to eliminate the blurring.

Several years ago a haploscope was devised by the writer[7] to demonstrate the behavior of the accommodative mechanism during the base in and base out duction tests. The instrument and results have been described elsewhere. The results show unmistakably that the blurring is a result of throwing the accommodative mechanism out of focus. It was further demonstrated that accommodation remains relatively constant from the base in to the base out blur point. At the base out blur point a rapid increase in accommodation occurs and at the base in blur point a rapid decrease. These changes in accommodation continue at the same rate all the way from the blur point to the break point. In the light of these demonstrations it is possible to describe the phenomena by saying that positive fusional convergence is used to bring the eyes from the phoria position to the base out to blur position and from this point on an increase in convergence is made possible only by increasing accommodation and taking advantage of the convergence associated with it. Negative fusional convergence is required to bring the eyes from the phoria position to the base in to blur position and from this point on accommodation must be decreased in order to obtain further divergence of the eyes. Usually no base in to blur point is found at 6 m. since accommodation is completely relaxed to start with. However, in cases of hypertonicity, where complete relaxation of accommodation is not attained in the binocular subjective test, a base in blur point at 6 m. may actually be encountered. In many individuals there are no base in and base out blur points at all. This condition is illustrated in Fig. 5. Such individuals simply do

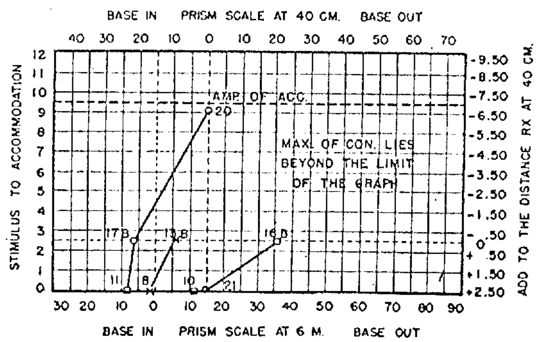

Fig. 5

not know the art of throwing the accommodative mechanism out of focus in order to gain an increase in convergence for the purpose of maintaining single vision. Some subjects are found to give base in blur points but not base out blur points or vice versa. The fundamental point to remember is that when the blur point does occur it represents the limit of fusional convergence, but if the break occurs before blur is obtained, the break point represents the limit of fusional convergence.

Significance of the Near Point of Convergence. At the near point of convergence, accommodative convergence and positive fusional convergence are both exerted to their limits and the intersection of the lines representing the amplitude of accommodation and the maximum of convergence may be taken as the limit of positive fusional convergence when accommodation is at its maximum.

[4] E. F. Tait, "A Report of Results of the Experimental Variation of the Stimulus Conditions in the Responses of the Accommodation Convergence Reflex," *Trans. of the Amer. Acad. of Optom.*, Vol. 7, pp. 199-206 (1932).

[5] E. S. Farmer, "Procedure for Analyzing Convergence and its Supporting Functions," *Trans. of the Amer. Acad. of Optom.*, Vol. 10, pp. 77-82 (1936).

[6] A. S. Percival, "The Relation of Convergence to Accommodation and Its Practical Bearing," *Ophthalmic Review*, Vol. 2, pp. 313-328 (1892).

[7] G. A. Fry, "An Experimental Analysis of the Accommodation-Convergence Relation," *Trans. of the Amer. Acad. of Optom.*, Vol. 11, pp. 64-76 (1937).

The Significance of Relative Accommodation. Starting with finite quantities of both accommodation and convergence it is possible to demonstrate a change in accommodation without a change in the position of the eyes but this does not mean that the innervation to convergence remains unchanged. As accommodation is increased the associated accommodative convergence increases. If there is exo. to start with this may change to an eso. which continues to increase. Eso. cases will show an increase. Such an increase in esophoria would produce diplopia were it not for negative fusional convergence which compensates the effect of the increase in accommodative convergence. When the negative fusional convergence is exhausted, no further increase in accommodative convergence is permissible without diplopia and hence the eyes fail to react to the blur produced by increasing the stimulus to accommodation. The minus lens to blur test is, therefore, an indirect method of determining the limit of negative fusional convergence.

If the A.C.A. ratio is small and the exo. at near is large, the stimulus to accommodation may be increased to the limit of the amplitude of accommodation without producing an esophoria which cannot be compensated, and consequently there is nothing to interfere with clear vision except the limit of accommodative amplitude. This type of case is illustrated in Fig. 5.

When 2.50 D. or slightly more needs to be added in the plus lens to blur test (see Fig. 6) this means that the exophoria produced by relaxing 2.50 D. of accommodation can be compensated by using positive fusional convergence. Whenever the plus lens to blur finding at 40 cm. is less than 2.50 D. it means that positive fusional convergence is exhausted before accommodation is completely relaxed, and in this case the

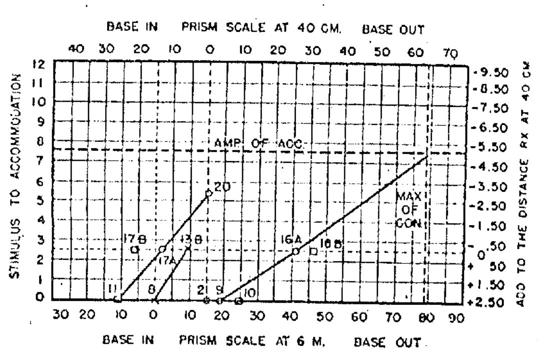

Fig. 6

plus lens to blur finding represents an indirect measure of the limit of positive fusional convergence.

The proposition that it is impossible to change accommodation without changing the innervation to convergence, is best demonstrated by the fact that the relative amplitude of accommodation is zero at the far and near points of convergence. At the far point of convergence accommodation cannot be inhibited because it is at the zero level already. It cannot be stimulated because if it were, accommodative convergence would come into play and the eyes could not be maintained in a position of maximum divergence.

For similar reasons the amplitude of relative accommodation must also be zero at the point of maximum convergence. At this point it is impossible to relax accommodation without getting a decrease in convergence, because positive fusional convergence which is already at its limit cannot supplement the loss in convergence associated with accommodation.

The investigation of the relative range of accommodation at the far and near points of convergence requires a more complete mapping of the limits of relative accommodation and convergence than is possible in a general examination. Numerous investigators have attempted to obtain complete sets of data and quite a number of cases have been reported in literature. Most of the cases indicate that the relative accommodation does become zero at both the near limit and the far limit of convergence. In a few cases, however, a certain amount of relative accommodation is manifest at one or the other of the limits of convergence. In the case described by Percival[8] there is indicated a relative range of accommodation at the far point of convergence. This investigator is the only one encountered whose data show this peculiarity. It may be due to some artifact but this is not apparent in the description of the experiment. The data published by many investigators such as Donders[9], Landolt[10] and Howe[11] do not extend beyond parallelism and hence throw no light on the problem of relative accommodation at the far point of convergence. The data obtained with haploscopes by

[8] A. S. Percival, *The Prescribing of Spectacles*, p. 127 (1928).

[9] F. C. Donders, *Accommodation and Refraction of the Eye* (Translated by W. D. Moore), pp. 110-126 (1864).

[10] E. Landolt, *Refraction and Accommodation of the Eye* (Translated by C. M. Culver), pp. 195-218 (1866).

[11] L. Howe, *The Muscles of the Eye*, Vol. 1, pp. 309-340 (1907).

Pereles[12], Hess[13] and the writer[14] all definitely show that the relative accommodation becomes zero at the far point of convergence.

The data presented by Donders, Landolt, Howe, Pereles and Percival demonstrate quite clearly that the relative range of accommodation becomes zero at the near point of convergence. There is only one case encountered so far in the literature which constitutes an exception to this rule. It is a case reported by Donders.

Construction of Lines on the Graph to Represent Limits of Positive and Negative Fusional Convergence with Various Amounts of Accommodation. It is helpful in the interpretation of the data to draw two lines on the graph, one connecting the various findings which represent limits of positive fusional convergence and another connecting the findings which represent limits of negative fusional convergence. Ordinarily the line representing the limit of positive fusional convergence is drawn by starting with the base out to blur at 6 m. (or break if there is no blur) and proceeding to the plus lens to blur, the base out to blur at 40 cm. (or break if there is no blur) and thence to the intersection of the lines representing maximum accommodation and maximum convergence. Whenever the base out to blur at 6 m. is more than 15△, the plus lens to blur does not represent a measure of the limit of positive fusional convergence and hence can be disregarded (See Fig. 6).

Ordinarily the line representing the limit of fusional convergence is drawn by starting with the base in to blur at 6 m. (or break if there is no blur) and proceeding to the base in to blur at 40 cm. (or break if there is no blur) and thence to the minus lens to blur at 40 cm. When the minus lens to blur finding extends all the way to the total amplitude of accommodation, it cannot represent a limit of negative fusional convergence, and consequently must be ignored.

The lines representing the limits of positive and negative fusional convergence are usually straight and parallel to each other. If this condition is not approximated the aberrant findings should be rechecked. Since these lines are also approximately parallel to the line connecting the phorias with various stimuli to accommodation, the slope of these lines can be used as a check on the A.C.A. ratio.

The Fundamental Variables. The area on the graph which includes all those combinations of stimuli to accommodation and convergence permitting clear and single vision, is bounded on the right hand side by the limit of positive fusional convergence, on the left hand side by the limit of negative fusional convergence, on the top by the maximum level of accommodation and on the bottom by the zero level. In general the shape of this area approximates a parallelogram. The line representing phoria findings runs approximately parallel to the two sides. These relationships are represented schematically in Fig. 7.

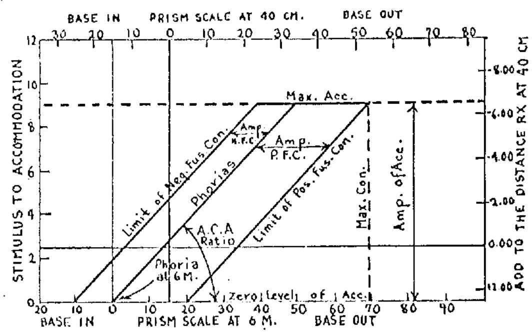

Fig. 7

The five fundamental variables in the relationship between accommodation and convergence may be listed as follows:

1. The phoria at distance, which determines the extent to which the whole parallelogram is displaced to the right or left.
2. The A.C.A. ratio which is given by the slope of the phoria line and also by the slope of the lines which represent the limits of positive and negative fusional convergence.
3. The amplitude of accommodation.
4. The amplitude of positive fusional convergence, which is represented by the lateral displacement of the P.F.C. limit from the phoria line.
5. The amplitude of negative fusional convergence, which is represented by the lateral displacement of the N.F.C. limit from the phoria line.

[12] H. Pereles, "Ueber die Relative Accommodations-breite," Graefe's *Arch. F. Ophthal.*, Vol. 35, pp. 84-115 (1889).
[13] L. Howe, Op. Cit., p. 332.
[14] G. A. Fry,, "Further Experiments on the Accommodation-Convergence Relationship," *Trans. of the Amer. Acad. of Optom.*, Vol. 12, pp. 65-74 (1938).

Indiana Sets Fall Seminar

The Third Annual IOA and AAO Fall Educational Seminar will be held at the Holiday Inn, Kokomo, Indiana on October 21.